

: fUi Ybdck Yf ZØf X]Y Dc`]h]_. A i ha UWX Yf]bbYb]a @UbXfUhgUa h < U» VYf[Y°



< U» VYf[Yž8Yi hgW `UbX'! =b' < U» Z fh'\Uh'Ua', "'*' `]'&\$&) 'X]Y
J YfUbghU`hi b[GdYYXXUh]b[ii bX'Dc`]h]_Yf]bbYb!HU`_ i]a
@UbXfUhgUa h'ghUhh[YZ bXYb"'N]Y`k Uf'Ygž'a Y\f': fUi Yb'ZØf'X]Y
?ca a i bU`dc`]h]_'ni '[Yk]bbYb"'Hfchn'XYf'K]WXh][_Y]h'k Y]V`]WXYf
Gh]a a Yb']b'XYf'Dc`]h]_'V`Y]Vh'XYf': fUi YbUbhY]``]b'j]Y`Yb'6YfY]WXYb
[Yf]b["'Bi f'nk Y]'6Øf[Yfa Y]ghYf]bbYb'g]bX']a '[YgUa hYb
@UbX_fY]g' < U» VYf[Y']b'&* 'GhAXhYb'i bX'; Ya Y]bXYb'j YfhfYhYbž'Y]b
5`Ufa nY]WXYb'ZØf'X]Y'dc`]h]gWXY'; 'Y]WXghY``i b["' < UggVYf[Y"XY
VYf]WXhYhž'XUgg'X]Y'ghY``j YfhfYhYbXY'7GI!DUfhY]j cfg]hnYbXY'i bX
6i bXYga]b]ghYf]b'8 cfch\YY'6Àf']b']\fYa'; fi »k cfh': fUi Yb'ni f
U_h]j Yb'A]h[YghU`hi b['Ui Zfi Zhii bX'g]Y'Yfa i h][hž']\fY'Gh]a a Y'ni</pre>

8 Yf + di h'XYf'Ua h]YfYbXYb'?ca a i bU`dc`]h]_Yf]bbYb'k Uf'YVYbgc k Yfhj c``k]Y'XYf'5i ghUi gWX'i bhYf'XYb'HY]`bY\a Yf]bbYb"'6]f[]h 6UmYfž'X]Y'ghY`j YfhfYhYbXY'@UbXfÀh]bž'VY[fØ» hY'X]Y HY]`bY\a Yf]bbYb'i bX'gWXi Z'gc'Y]bY'dcg]h]j Y'6Ug]g'ZØf'X]Y 8]g_i gg]cb"'5a '; YgdfÀWX'VYhY]`][h'k UfYb'Ui WX'***`]hhU'C hhž'X]Y 8 f]hhY'6Øf[Yfa Y]ghYf]b'j cb'<cZ\Y]a ž'i bX'8Ub]Y`U'@Ub[ž'X]Y Nk Y]hY'6Øf[Yfa Y]ghYf]b'j cb'K cbZ fhž'X]Y']\fY'9fZU\fi b[Yb']b'XYf Dc`]h]_'hY]`hYb"'+b'Y]bYf'`cW_YfYb'5ha cgd\ÀfY'k i fXYb'X]Y < YfUi gZcfXYfi b[Yb'i bX'7\UbW'b'XYg'dc`]h]gWXYb'9b[U[Ya Ybhg h\Ya Uh]q]Yfh"

Dc`]h]_ i bX : Ua]`]Y

9]b 'nYbhfu`Yg`H\Ya U`k Uf`X]Y`J YfY]bVUf_Y]h'j cb`: Ua]`]Yž'6Yfi Z i bX`dc`]h]gW\Ya `9b[U[Ya Ybh"`I `f]_Y`GW\Y``YbVYf[Yfž') , `\$J\fY U`hž\Ai » YfhY`g]W\`ØVYf`]\fY`6YXYb_Ybž'YfbYi h'ZØf`XYb`GhUXhfUh`ni _UbX]X]YfYb"`7`Ui X]U`; `ØW_Yfhž'GhUXhf\Ah]b`]b`< U» Zi fhž'Yf`\Ai hYfhY XYb`NY]h!`i bX`9b[U[Ya YbhUi Zk UbX`Y]bYg`A UbXUhg`i bX``]YZYfhYb gc`XYb`5bk YgYbXYb`dfU_h]gW\Y`9]bV`]\\Y\"`5i W\`\\$ X]h\`; Y]`]b[ž GhUXhf\Ah]b`Ui g`< cZ\Y]a ž`Yfa i h][hY`X]Y`: fUi Yb`ni f`? UbX]XUhi f i bX`VYhcbhYž`XUgg`: fUi Yb`Yb[U[]Yfh`i bX`_ca dYhYbh`gY]Yb"

9fZfYi`]W\ni \\OfYb]ghžXUggX]YDUfhY]ni [Y\Of][_Y]hVY]XYf

J YfUbghU`hi b['_Y]bY'Fc``Ygd]Y`hY"8Yf'5i ghUi gW\k Uf'cZZYb'i bX
dUfhY]ØVYf[fY]ZYbX": fUi Yb'VYfY]W\Yfb'X]Y'Dc`]h]_ ž'gc'VYhcbhY

i`]hhU'Chhi bX'ZUbX'XUa]h'[fc>Yb'5b_`Ub[']a'Di V`]_i a'''

6i bXYggh]Zhi b['; `Y]W\ghY``i b['ZØ\fh'Ui gž'XUgg'XYf

Xi fW\gW\b]hh`]W\Y': fUi YbUbhY]`']b'XYb'_ca a i bU`Yb'JYfhfYhi b[YbYhk U'' \$ž) i 'VYhfÀ[h"8Yf': fUi YbUbhY]`'ghU[b]Yfh'gY]h'a Y\f'U`g'&\$

J\fYb']a '8Yi hgW\Yb'6i bXYghU['VY]'fi bX'Y]bYa '8f]hhY`ž'k Ug

XYi h`]W\'a UW\hž'XUgg'\]Yf'bcW\'j]Y`'DchYbn]U`'j cf\UbXYb']gh"

8]Y'[`ÀgYfbY'8YW_Y'VfYWXYb

6YgcbXYfg']b'\O\YfYb'9bhgW\Y]Xi b[gYVYbYbž'k]Y'XYb'@UbXYg! i bX'6i bXYgdUf`Ua YbhYbž']gh'X]Y': fUi Ybj YfhfYhi b['[Yf]b["'

: fUi Yb"J]Y`ZU`h"Dc`]h]_ `nY][h`Ui Zž`XUgg`]b`XYb
6i bXYghU[gZfU_h]cbYb`XYf`\ ÒW\ghY`: fUi YbUbhY]``VY]`6ØbXb]g
- \$#8]Y`; fØbYb`a]h`* %ž&`i ``]Y[hž`[YZc`[h'j cb`XYf`@-B? 9B`a]h`) * ž&
i `i bX`XYf`GD8`a]h`(%ž+`i "'-a `; Y[YbgUhn`XUni `YffY]W\Yb`X]Y
7GI `i bX`78I `bi f`&) `i `Vnk "'&&ž*`i "'8]Y`5Z8`\Uh`a]h`%%ž, `i `XYb
b]YXf][ghYb`: fUi YbUbhY]`ž`k Ug`X]Y`XYfnY]h][Y`I b[`Y]W\\Y]h
i bhYfghfY]W\h"

 $\label{thm:converse} \begin{align*} I a `X]Y`[``AgYfbY`8YW_Y`ni `Xi fW\VfYW\Ybž`g]bX`j YfV]bX`]W\Y \\ E i chYbž`[Yn]Y`hY`: OfXYfi b[`i bX`dUfhY]]bhYfbY`FYZcfa Yb \\ bchk YbX]["`9g`]gh`YbhgW\Y]XYbXž`XUgg`U``Y`6YhY]`][hYb`U_h]j \\ UfVY]hYbž`i a `X]Y`FYdfAgYbhUh]cb`j cb`: fUi Yb`]b`XYf`Dc`]h]_`ni \\ ghY][Yfb`i bX`Ui Z`Y]b`Ui g[Yk c[YbYfYg`A UW\hj Yf\A`hb]g \\ \]bni UfVY]hYb"`8]Y`<U» Zi fhYf`JYfUbghU`hi b[`]gh`Y]b`k]W\h][Yf GW\f]hh`]b`X]Y`f]W\h][Y`F]W\hi b[`ži a `: fUi Yb`ni `a ch]j]YfYbž`g]W\u_h]j `]b`XYf`Dc`]h]_`ni `Yb[U[]YfYb`i bX`ZØf`]\fY`FYW\hY Y]bni hfYhYb"$

8 YhU]`g	
Cfh	< U» VYf[Yž'8Yi hgW\`UbX
EiY``Yb	• k k k "\UggVYf[Y"XY
	• k k k "Vi b X Y ggh]Zhi b [!
	[`Y]W\ ghY ``i b["XY
	ZfUi Yb!j]Y`ZU`h!dc`]h]_"XY

6Ygi WXYb'G]Y'i bg'Ui Z.'bU[!VUmYfb"XY